Cambridge Assessment
International Education

Syllabus

Cambridge International AS & A Level Mathematics 9709

For examination in June and November 2020, 2021 and 2022.
Also available for examination in March 2020, 2021 and 2022 for India only.

Version 1

Cambridge Pathway

Why choose Cambridge?

Cambridge Assessment International Education prepares school students for life, helping them develop an informed curiosity and a lasting passion for learning. We are part of the University of Cambridge.

Our international qualifications are recognised by the world's best universities and employers, giving students a wide range of options in their education and career. As a not-for-profit organisation, we devote our resources to delivering high-quality educational programmes that can unlock learners' potential.

Our programmes and qualifications set the global standard for international education. They are created by subject experts, rooted in academic rigour and reflect the latest educational research. They provide a strong platform for students to progress from one stage to the next, and are well supported by teaching and learning resources.

We review all our syllabuses regularly, so they reflect the latest research evidence and professional teaching practice – and take account of the different national contexts in which they are taught.

We consult with teachers to help us design each syllabus around the needs of their learners. Consulting with leading universities has helped us make sure our syllabuses encourage students to master the key concepts in the subject and develop the skills necessary for success in higher education.

Our mission is to provide educational benefit through provision of international programmes and qualifications for school education and to be the world leader in this field. Together with schools, we develop Cambridge learners who are confident, responsible, reflective, innovative and engaged – equipped for success in the modern world.

Every year, nearly a million Cambridge students from 10 000 schools in 160 countries prepare for their future with an international education from Cambridge International.

'We think the Cambridge curriculum is superb preparation for university.'

Christoph Guttentag, Dean of Undergraduate Admissions, Duke University, USA

Quality management

Our systems for managing the provision of international qualifications and education programmes for students aged 5 to 19 are certified as meeting the internationally recognised standard for quality management, ISO 9001:2008. Learn more at www.cambridgeinternational.org/ISO9001

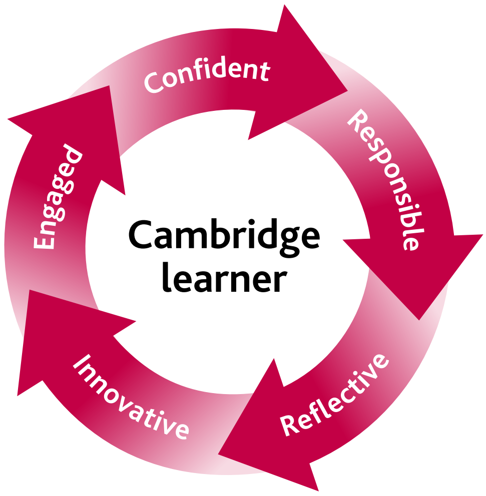
Contents

Changes to this syllabus

For information about changes to this syllabus for 2020, 2021 and 2022, go to page 53.

1 Why choose this syllabus?

Key benefits



The best motivation for a student is a real passion for the subject they're learning. By offering students a variety of Cambridge International AS & A Levels, you can give them the greatest chance of finding the path of education they most want to follow. With over 50 subjects to choose from, students can select the ones they love and that they're best at, which helps motivate them throughout their studies.

Following a Cambridge International AS & A Level programme helps students develop abilities which universities value highly, including:

- a deep understanding of their subjects
- higher order thinking skills – analysis, critical thinking, problem solving
- presenting ordered and coherent arguments
- independent learning and research.

Cambridge International AS & A Level Mathematics develops a set of transferable skills. These include the skill of working with mathematical information, as well as the ability to think logically and independently, consider accuracy, model situations mathematically, analyse results and reflect on findings. Learners can apply these skills across a wide range of subjects and the skills equip them well for progression to higher education or directly into employment.

Our approach in Cambridge International AS & A Level Mathematics encourages learners to be:

confident, using and sharing information and ideas, and using mathematical techniques to solve problems. These skills build confidence and support work in other subject areas as well as in mathematics.

responsible, through learning and applying skills which prepare them for future academic studies, helping them to become numerate members of society.

reflective, through making connections between different branches of mathematics and considering the outcomes of mathematical problems and modelling.

innovative, through solving both familiar and unfamiliar problems in different ways, selecting from a range of mathematical and problem-solving techniques.

engaged, by the beauty and structure of mathematics, its patterns and its many applications to real life situations.

'Cambridge students develop a deep understanding of subjects and independent thinking skills.'

Tony Hines, Principal, Rockledge High School, USA

Key concepts

Key concepts are essential ideas that help students develop a deep understanding of their subject and make links between different aspects. Key concepts may open up new ways of thinking about, understanding or interpreting the important things to be learned.

Good teaching and learning will incorporate and reinforce a subject's key concepts to help students gain:

- a greater depth as well as breadth of subject knowledge
- confidence, especially in applying knowledge and skills in new situations
- the vocabulary to discuss their subject conceptually and show how different aspects link together
- a level of mastery of their subject to help them enter higher education.

The key concepts identified below, carefully introduced and developed, will help to underpin the course you will teach. You may identify additional key concepts which will also enrich teaching and learning.

The key concepts for Cambridge International AS & A Level Mathematics are:

- Problem solving
  Mathematics is fundamentally problem solving and representing systems and models in different ways. These include:
  - Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
  - Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
  - Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
  - Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
  - Statistical methods: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

- Communication
  Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.

- Mathematical modelling
  Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

Recognition and progression

Every year thousands of students with Cambridge International AS & A Levels gain places at leading universities worldwide. Cambridge International AS & A Levels are accepted across 195 countries. They are valued by top universities around the world including those in the UK, US (including Ivy League universities), Europe, Australia, Canada and New Zealand.

UK NARIC, the national agency in the UK for the recognition and comparison of international qualifications and skills, has carried out an independent benchmarking study of Cambridge International AS & A Level and found it to be comparable to the standard of AS & A Level in the UK. This means students can be confident that their Cambridge International AS & A Level qualifications are accepted as equivalent, grade for grade, to UK AS & A Levels by leading universities worldwide.

Cambridge International AS Level Mathematics makes up the first half of the Cambridge International A Level course in mathematics and provides a foundation for the study of mathematics at Cambridge International A Level. Depending on local university entrance requirements, students may be able to use it to progress directly to university courses in mathematics or some other subjects. It is also suitable as part of a course of general education.

Cambridge International A Level Mathematics provides a foundation for the study of mathematics or related courses in higher education. Equally it is suitable as part of a course of general education.

For more information about the relationship between the Cambridge International AS Level and Cambridge International A Level see the 'Assessment overview' section of the Syllabus overview.

We recommend learners check the Cambridge recognitions database and the university websites to find the most up-to-date entry requirements for courses they wish to study.

Learn more at **www.cambridgeinternational.org/recognition**

'The depth of knowledge displayed by the best A Level students makes them prime targets for America's Ivy League universities'

Yale University, USA

Supporting teachers

We provide a wide range of practical resources, detailed guidance, and innovative training and professional development so that you can give your learners the best possible preparation for Cambridge International AS & A Level.

Support for Cambridge International AS & A Level

Teaching resources

- The School Support Hub **www.cambridgeinternational.org/support**
- Syllabus
- Scheme of work
- Learner guide
- Discussion forum
- Resource list
- Endorsed textbooks and digital resources

Exam preparation resources

- Question papers
- Mark schemes
- Example candidate responses to understand what examiners are looking for at key grades
- Examiner reports to improve future teaching

Training

- Face-to-face workshops around the world
- Online self-study training
- Online tutor-led training
- Cambridge Professional Development Qualifications

Community

You can find useful information, as well as share your ideas and experiences with other teachers, on our social media channels and community forums.

Find out more at **www.cambridgeinternational.org/social-media**

'Cambridge International AS & A Levels prepare students well for university because they've learnt to go into a subject in considerable depth. There's that ability to really understand the depth and richness and the detail of a subject. It's a wonderful preparation for what they are going to face at university.'

US Higher Education Advisory Council

2 Syllabus overview

Aims

The aims describe the purposes of a course based on this syllabus.

The aims are to enable students to:

- develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment
- develop an understanding of mathematical principles and an appreciation of mathematics as a logical and coherent subject
- acquire a range of mathematical skills, particularly those which will enable them to use applications of mathematics in the context of everyday situations and of other subjects they may be studying
- develop the ability to analyse problems logically
- recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and select an appropriate mathematical method to solve the problem
- use mathematics as a means of communication with emphasis on the use of clear expression
- acquire the mathematical background necessary for further study in mathematics or related subjects.

Support for Cambridge International AS & A Level Mathematics

Our School Support Hub **www.cambridgeinternational.org/support** provides Cambridge schools with a secure site for downloading specimen and past question papers, mark schemes, grade thresholds and other curriculum resources specific to this syllabus. The School Support Hub community offers teachers the opportunity to connect with each other and to ask questions related to the syllabus.

Content overview

Content section	Assessment component	Topics included
1 Pure Mathematics 1	Paper 1	1.1 Quadratics 1.2 Functions 1.3 Coordinate geometry 1.4 Circular measure 1.5 Trigonometry 1.6 Series 1.7 Differentiation 1.8 Integration
2 Pure Mathematics 2	Paper 2	2.1 Algebra 2.2 Logarithmic and exponential functions 2.3 Trigonometry 2.4 Differentiation 2.5 Integration 2.6 Numerical solution of equations
3 Pure Mathematics 3	Paper 3	3.1 Algebra 3.2 Logarithmic and exponential functios 3.3 Trigonometry 3.4 Differentiation 3.5 Integration 3.6 Numerical solution of equations 3.7 Vectors 3.8 Differential equations 3.9 Complex numbers
4 Mechanics	Paper 4	4.1 Forces and equilibrium 4.2 Kinematics of motion in a straight line 4.3 Momentum 4.4 Newton's laws of motion 4.5 Energy, work and power
5 Probability & Statistics 1	Paper 5	5.1 Representation of data 5.2 Permutations and combinations 5.3 Probability 5.4 Discrete random variables 5.5 The normal distribution
6 Probability & Statistics 2	Paper 6	6.1 The Poisson distribution 6.2 Linear combinations of random variables 6.3 Continuous random variables 6.4 Sampling and estimation 6.5 Hypothesis tests

Structure

There are six components that can be combined in specific ways (please see below for details):

Paper 1: Pure Mathematics 1

Paper 2: Pure Mathematics 2

Paper 3: Pure Mathematics 3

Paper 4: Mechanics

Paper 5: Probability & Statistics 1

Paper 6: Probability & Statistics 2

All AS Level candidates take two written papers.

All A Level candidates take four written papers.

AS Level Mathematics

The Cambridge International AS Level Mathematics qualification offers three different options:

- Pure Mathematics only (Paper 1 and Paper 2) **or**
- Pure Mathematics and Mechanics (Paper 1 and Paper 4) **or**
- Pure Mathematics and Probability & Statistics (Paper 1 and Paper 5).

Please note, the Pure-Mathematics-only option (Paper 1 and Paper 2) leads to an AS Level only and cannot be used as a staged route to a full A Level. Candidates who have taken Paper 1 and Paper 2 at AS Level and then wish to complete a full A Level would need to retake Paper 1 alongside three other components. These should be chosen from the specific combinations below.

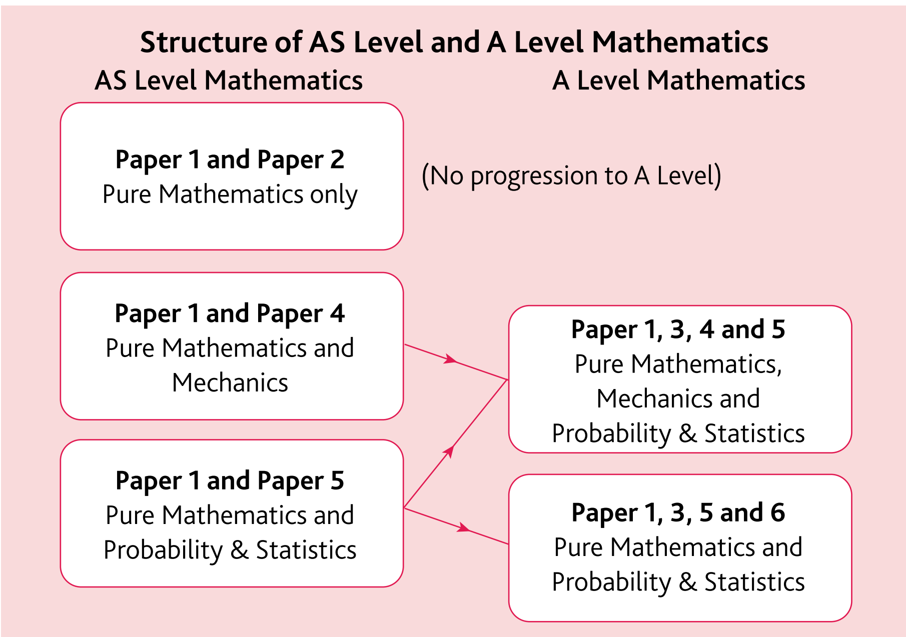
A Level Mathematics

The Cambridge International A Level Mathematics qualification offers two different options:

- Pure Mathematics, Mechanics and Probability & Statistics (Papers 1, 3, 4 and 5) **or**
- Pure Mathematics and Probability & Statistics (Papers 1, 3, 5 and 6).

Please note, it is not possible to combine Paper 4 and Paper 6. This is because Paper 6 depends on prior knowledge of the subject content for Paper 5.

See page 10 for a table showing all possible assessment routes.



Assessment overview

Paper 1

Pure Mathematics 1 — 1 hour 50 minutes
75 marks
10 to 12 structured questions based on the Pure Mathematics 1 subject content
Written examination
Externally assessed
60% of the AS Level
30% of the A Level
Compulsory for AS Level and A Level

Paper 2

Pure Mathematics 2 — 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the Pure Mathematics 2 subject content
Written examination
Externally assessed
40% of the AS Level
Offered only as part of AS Level

Paper 3

Pure Mathematics 3 — 1 hour 50 minutes
75 marks
9 to 11 structured questions based on the Pure Mathematics 3 subject content
Written examination
Externally assessed
30% of the A Level only
Compulsory for A Level

Paper 4

Mechanics — 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the Mechanics subject content
Written examination
Externally assessed
40% of the AS Level
20% of the A Level
Offered as part of AS Level or A Level

Paper 5

Probability & Statistics 1 — 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the Probability & Statistics 1 subject content
Written examination
Externally assessed
40% of the AS Level
20% of the A Level
Compulsory for A Level

Paper 6

Probability & Statistics 2 — 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the Probability & Statistics 2 subject content
Written examination
Externally assessed
20% of the A Level only
Offered only as part of A Level

Three routes for Cambridge International AS & A Level Mathematics

Candidates may combine components as shown below to suit their particular interests.

<table>
<tr><th>Route 1
AS Level only
(Candidates take the AS components in the same series)</th><th>Paper 1
Pure Mathematics 1</th><th>Paper 2
Pure Mathematics 2</th><th>Paper 3
Pure Mathematics 3</th><th>Paper 4
Mechanics</th><th>Paper 5
Probability & Statistics 1</th><th>Paper 6
Probability & Statistics 2</th></tr>
<tr><td>Either</td><td>✓</td><td></td><td rowspan="3">Not available for AS Level</td><td></td><td>✓</td><td rowspan="3">Not available for AS Level</td></tr>
<tr><td>Or</td><td>✓</td><td></td><td>✓</td><td></td></tr>
<tr><td>Or
Note this option in Route 1 cannot count towards A Level</td><td>✓</td><td>✓</td><td></td><td></td></tr>
</table>

<table>
<tr><th>Route 2
A Level (staged over two years)</th><th>Paper 1
Pure Mathematics 1</th><th>Paper 2
Pure Mathematics 2</th><th>Paper 3
Pure Mathematics 3</th><th>Paper 4
Mechanics</th><th>Paper 5
Probability & Statistics 1</th><th>Paper 6
Probability & Statistics 2</th></tr>
<tr><td>Either
Year 1 AS Level</td><td>✓</td><td rowspan="6">Not available for A Level</td><td></td><td>✓</td><td></td><td></td></tr>
<tr><td>Year 2 Complete the A Level</td><td></td><td>✓</td><td></td><td>✓</td><td></td></tr>
<tr><td>Or
Year 1 AS Level</td><td>✓</td><td></td><td></td><td>✓</td><td></td></tr>
<tr><td>Year 2 Complete the A Level</td><td></td><td>✓</td><td></td><td></td><td>✓</td></tr>
<tr><td>Or
Year 1 AS Level</td><td>✓</td><td></td><td></td><td>✓</td><td></td></tr>
<tr><td>Year 2 Complete the A Level</td><td></td><td>✓</td><td>✓</td><td></td><td></td></tr>
</table>

<table>
<tr><th>Route 3
A Level
(Candidates take the A Level components in the same series)</th><th>Paper 1
Pure Mathematics 1</th><th>Paper 2
Pure Mathematics 2</th><th>Paper 3
Pure Mathematics 3</th><th>Paper 4
Mechanics</th><th>Paper 5
Probability & Statistics 1</th><th>Paper 6
Probability & Statistics 2</th></tr>
<tr><td>Either</td><td>✓</td><td rowspan="2">Not available for A Level</td><td>✓</td><td>✓</td><td>✓</td><td></td></tr>
<tr><td>Or</td><td>✓</td><td>✓</td><td></td><td>✓</td><td>✓</td></tr>
</table>

Assessment objectives

The assessment objectives (AOs) are:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weighting for assessment objectives

The approximate weightings (± 5%) allocated to each of the assessment objectives (AOs) are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

3 Subject content

The mathematical content for each component is detailed below. You can teach the topics in any order you find appropriate.

Information about calculator use and information about the relationships between syllabus components can be found in 4 Details of the assessment.

Notes and examples are included to clarify the subject content. Please note that these are examples only and examination questions may differ from the examples given.

Prior knowledge

Knowledge of the content of the Cambridge IGCSE® Mathematics 0580 (Extended curriculum), or Cambridge International O Level (4024/4029), is assumed. Candidates should be familiar with scientific notation for compound units, e.g. $5\,\mathrm{m\,s^{-1}}$ for 5 metres per second.

In addition, candidates should:

- be able to carry out simple manipulation of surds (e.g. expressing $\sqrt{12}$ as $2\sqrt{3}$ and $\frac{6}{\sqrt{2}}$ as $3\sqrt{2}$),
- know the shapes of graphs of the form $y = kx^n$, where k is a constant and n is an integer (positive or negative) or $\pm\frac{1}{2}$.

1 Pure Mathematics 1 (for Paper 1)

1.1 Quadratics

Candidates should be able to:	Notes and examples
• carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form	e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph
• find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant	e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term ‘repeated root’ is included.
• solve quadratic equations, and quadratic inequalities, in one unknown	By factorising, completing the square and using the formula.
• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic	e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$, $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.
• recognise and solve equations in x which are quadratic in some function of x.	e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$, $\tan^2 x = 1 + \tan x$.

1 Pure Mathematics 1

1.2 Functions

Candidates should be able to:	Notes and examples
• understand the terms function, domain, range, one-one function, inverse function and composition of functions	
• identify the range of a given function in simple cases, and find the composition of two given functions	e.g. range of $\mathrm{f}: x \mapsto \frac{1}{x}$ for $x \geqslant 1$ and range of $\mathrm{g}: x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g.
• determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases	e.g. finding the inverse of $\mathrm{h}: x \mapsto (2x+3)^2 - 4$ for $x < -\frac{3}{2}$.
• illustrate in graphical terms the relation between a one-one function and its inverse	Sketches should include an indication of the mirror line $y = x$.
• understand and use the transformations of the graph of $y = \mathrm{f}(x)$ given by $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(ax)$ and simple combinations of these.	Including use of the terms 'translation', 'reflection' and 'stretch' in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

1.3 Coordinate geometry

Candidates should be able to:	Notes and examples
• find the equation of a straight line given sufficient information	e.g. given two points, or one point and the gradient.
• interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems	Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.
• understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r	Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.
• use algebraic methods to solve problems involving lines and circles	Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.
• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.	e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

1 Pure Mathematics 1

1.4 Circular measure

Candidates should be able to:	Notes and examples
• understand the definition of a radian, and use the relationship between radians and degrees	
• use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.	Including calculation of lengths and angles in triangles and areas of triangles.

1.5 Trigonometry

Candidates should be able to:	Notes and examples
• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)	Including e.g. $y = 3\sin x$, $y = 1 - \cos 2x$, $y = \tan\left(x + \frac{1}{4}\pi\right)$.
• use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles	e.g. $\cos 150^\circ = -\frac{1}{2}\sqrt{3}$, $\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$.
• use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations	No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected.
• use the identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$	e.g. in proving identities, simplifying expressions and solving equations.
• find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).	e.g. solve $3\sin 2x + 1 = 0$ for $-\pi < x < \pi$, $3\sin^2\theta - 5\cos\theta - 1 = 0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$.

1.6 Series

Candidates should be able to:	Notes and examples
• use the expansion of $(a + b)^n$, where n is a positive integer	Including the notations $\binom{n}{r}$ and $n!$ Knowledge of the greatest term and properties of the coefficients are not required.
• recognise arithmetic and geometric progressions	
• use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions	Including knowledge that numbers a, b, c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent). Questions may involve more than one progression.
• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.	

1 Pure Mathematics 1

1.7 Differentiation

Candidates should be able to:	Notes and examples
• understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives	Only an informal understanding of the idea of a limit is expected. e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2 + h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.
• use the derivative of x^n (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule	e.g. find $\frac{dy}{dx}$, given $y = \sqrt{2x^3 + 5}$.
• apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change	Including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.
• locate stationary points and determine their nature, and use information about stationary points in sketching graphs.	Including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified. Knowledge of points of inflexion is not included.

1.8 Integration

Candidates should be able to:	Notes and examples
• understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except –1), together with constant multiples, sums and differences	e.g. $\int (2x^3 - 5x + 1)\,dx$, $\int \frac{1}{(2x+3)^2}\,dx$.
• solve problems involving the evaluation of a constant of integration	e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x+1}$.
• evaluate definite integrals	Including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}}\,dx$ and $\int_1^\infty x^{-2}\,dx$.
• use definite integration to find – the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves – a volume of revolution about one of the axes.	A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x-axis.

2 Pure Mathematics 2 (for Paper 2)

Knowledge of the content for Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

2.1 Algebra

Candidates should be able to:	Notes and examples
• understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities	Graphs of $y = |f(x)|$ and $y = f(|x|)$ for non-linear functions f are not included. e.g. $|3x - 2| = |2x + 7|$, $2x + 5 < |x + 1|$
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	
• use the factor theorem and the remainder theorem.	e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

2.2 Logarithmic and exponential functions

Candidates should be able to:	Notes and examples
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	
• understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs	Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k.
• use logarithms to solve equations and inequalities in which the unknown appears in indices	e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.	e.g. $y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$ $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

2 Pure Mathematics 2

2.3 Trigonometry

Candidates should be able to:	Notes and examples
• understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude	
• use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of – $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.	e.g. simplifying $\cos(x - 30°) - 3\sin(x - 60°)$. e.g. solving $\tan\theta + \cot\theta = 4$, $2\sec^2\theta - \tan\theta = 5$, $3\cos\theta + 2\sin\theta = 1$.

2.4 Differentiation

Candidates should be able to:	Notes and examples
• use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites	
• differentiate products and quotients	e.g. $\dfrac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2}.
• find and use the first derivative of a function which is defined parametrically or implicitly.	e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.

2.5 Integration

Candidates should be able to:	Notes and examples
• extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\dfrac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$	Knowledge of the general method of integration by substitution is not required.
• use trigonometrical relationships in carrying out integration	e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.
• understand and use the trapezium rule to estimate the value of a definite integral.	Including use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.

2 Pure Mathematics 2

2.6 Numerical solution of equations

Candidates should be able to:	Notes and examples
• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change	e.g. finding a pair of consecutive integers between which a root lies.
• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	
• understand how a given simple iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.	Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3 Pure Mathematics 3 (for Paper 3)

Knowledge of the content of Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

3.1 Algebra

Candidates should be able to:	Notes and examples
• understand the meaning of $\lvert x\rvert$, sketch the graph of $y = \lvert ax + b\rvert$ and use relations such as $\lvert a\rvert = \lvert b\rvert \Leftrightarrow a^2 = b^2$ and $\lvert x - a\rvert < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities	Graphs of $y = \lvert \mathrm{f}(x)\rvert$ and $y = \mathrm{f}(\lvert x\rvert)$ for non-linear functions f are not included. e.g. $\lvert 3x - 2\rvert = \lvert 2x + 7\rvert$, $2x + 5 < \lvert x + 1\rvert$.
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	
• use the factor theorem and the remainder theorem	e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients. Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.
• recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than – $(ax + b)(cx + d)(ex + f)$ – $(ax + b)(cx + d)^2$ – $(ax + b)(cx^2 + d)$	Excluding cases where the degree of the numerator exceeds that of the denominator
• use the expansion of $(1 + x)^n$, where n is a rational number and $\lvert x\rvert < 1$.	Finding the general term in an expansion is not included. Adapting the standard series to expand e.g. $\left(2 - \frac{1}{2}x\right)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.

3 Pure Mathematics 3

3.2 Logarithmic and exponential functions

Candidates should be able to:	Notes and examples
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	
• understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs	Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k.
• use logarithms to solve equations and inequalities in which the unknown appears in indices	e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.	e.g. $y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$. $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

3.3 Trigonometry

Candidates should be able to:	Notes and examples
• understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude	
• use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of – $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.	e.g. simplifying $\cos(x - 30°) - 3\sin(x - 60°)$. e.g. solving $\tan\theta + \cot\theta = 4$, $2\sec^2\theta - \tan\theta = 5$, $3\cos\theta + 2\sin\theta = 1$.

3 Pure Mathematics 3

3.4 Differentiation

Candidates should be able to:	Notes and examples
• use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, together with constant multiples, sums, differences and composites	Derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are not required.
• differentiate products and quotients	e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2}.
• find and use the first derivative of a function which is defined parametrically or implicitly.	e.g. $x = t - e^{2t}$, $y = t + e^{2t}$. e.g. $x^2 + y^2 = xy + 7$. Including use in problems involving tangents and normals.

3.5 Integration

Candidates should be able to:	Notes and examples
• extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\frac{1}{x^2+a^2}$	Including examples such as $\frac{1}{2+3x^2}$.
• use trigonometrical relationships in carrying out integration	e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.
• integrate rational functions by means of decomposition into partial fractions	Restricted to types of partial fractions as specified in topic 3.1 above.
• recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate such functions	e.g. integration of $\frac{x}{x^2+1}$, $\tan x$.
• recognise when an integrand can usefully be regarded as a product, and use integration by parts	e.g. integration of $x \sin 2x$, $x^2 e^{-x}$, $\ln x$, $x \tan^{-1} x$.
• use a given substitution to simplify and evaluate either a definite or an indefinite integral.	e.g. to integrate $\sin^2 2x \cos x$ using the substitution $u = \sin x$.

3 Pure Mathematics 3

3.6 Numerical solution of equations

Candidates should be able to:	Notes and examples
• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change	e.g. finding a pair of consecutive integers between which a root lies.
• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation	
• understand how a given simple iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.	Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3.7 Vectors

Candidates should be able to:	Notes and examples
• use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$	
• carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms	e.g. '$OABC$ is a parallelogram' is equivalent to $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$. The general form of the ratio theorem is not included, but understanding that the midpoint of AB has position vector $\frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$ is expected.
• calculate the magnitude of a vector, and use unit vectors, displacement vectors and position vectors	In 2 or 3 dimensions.
• understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information	e.g. finding the equation of a line given the position vector of a point on the line and a direction vector, or the position vectors of two points on the line.
• determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists	Calculation of the shortest distance between two skew lines is not required. Finding the equation of the common perpendicular to two skew lines is also not required.
• use formulae to calculate the scalar product of two vectors, and use scalar products in problems involving lines and points.	e.g. finding the angle between two lines, and finding the foot of the perpendicular from a point to a line; questions may involve 3D objects such as cuboids, tetrahedra (pyramids), etc. Knowledge of the vector product is not required.

3 Pure Mathematics 3

3.8 Differential equations

Candidates should be able to:	Notes and examples
• formulate a simple statement involving a rate of change as a differential equation	The introduction and evaluation of a constant of proportionality, where necessary, is included.
• find by integration a general form of solution for a first order differential equation in which the variables are separable	Including any of the integration techniques from topic 3.5 above.
• use an initial condition to find a particular solution	
• interpret the solution of a differential equation in the context of a problem being modelled by the equation.	Where a differential equation is used to model a 'real-life' situation, no specialised knowledge of the context will be required.

3.9 Complex numbers

Candidates should be able to:	Notes and examples
• understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal	Notations $\mathrm{Re}\,z$, $\mathrm{Im}\,z$, $\lvert z\rvert$, $\arg z$, z^* should be known. The argument of a complex number will usually refer to an angle θ such that $-\pi < \theta \leqslant \pi$, but in some cases the interval $0 \leqslant \theta < 2\pi$ may be more convenient. Answers may use either interval unless the question specifies otherwise.
• carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in Cartesian form $x + \mathrm{i}y$	For calculations involving multiplication or division, full details of the working should be shown.
• use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs	e.g. in solving a cubic or quartic equation where one complex root is given.
• represent complex numbers geometrically by means of an Argand diagram	
• carry out operations of multiplication and division of two complex numbers expressed in polar form $r(\cos\theta + \mathrm{i}\sin\theta) \equiv r\mathrm{e}^{\mathrm{i}\theta}$	Including the results $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$, and corresponding results for division.
• find the two square roots of a complex number	e.g. the square roots of $5 + 12\mathrm{i}$ in exact Cartesian form. Full details of the working should be shown.
• understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers	
• illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram	e.g. $\lvert z - a\rvert < k$, $\lvert z - a\rvert = \lvert z - b\rvert$, $\arg(z - a) = \alpha$.

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$\sin(90^\circ - \theta) \equiv \cos\theta$, $\cos(90^\circ - \theta) \equiv \sin\theta$, $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta \equiv 1$.

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram.
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing.
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact, and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.
• use Newton's third law.	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:	Notes and examples
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities	Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration	
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration	Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.
• use appropriate formulae for motion with constant acceleration in a straight line.	Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:	Notes and examples
• use the definition of linear momentum and show understanding of its vector nature	For motion in one dimension only.
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.	Including direct impact of two bodies where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:	Notes and examples
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod	If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.
• use the relationship between mass and weight	$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value 10 (m s^{-2}) for g is expected.
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration	Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.
• solve simple problems which may be modelled as the motion of connected particles.	e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:	Notes and examples
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force	$W = Fd\cos\theta$; Use of the scalar product is not required.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy	Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion	Including calculation of (average) power as $\frac{\text{Work done}}{\text{Time taken}}$. $P = Fv$.
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $\mathrm{P}(A)$, $\mathrm{P}(A \cup B)$, $P(A \cap B)$, $\mathrm{P}(A|B)$ and the use of A' to denote the complement of A.

5.1 Representation of data

Candidates should be able to:	Notes and examples
• select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have	
• draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs	Including back-to-back stem-and-leaf diagrams.
• understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)	e.g. in comparing and contrasting sets of data.
• use a cumulative frequency graph	e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.
• calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals Σx and Σx^2, or coded totals $\Sigma(x-a)$ and $\Sigma(x-a)^2$, and use such totals in solving problems which may involve up to two data sets.	

5.2 Permutations and combinations

Candidates should be able to:	Notes and examples
• understand the terms permutation and combination, and solve simple problems involving selections	
• solve problems about arrangements of objects in a line, including those involving – repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') – restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).	Questions may include cases such as people sitting in two (or more) rows. Questions about objects arranged in a circle will not be included.

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:	Notes and examples
• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations	e.g. the total score when two fair dice are thrown. e.g. drawing balls at random from a bag containing balls of different colours.
• use addition and multiplication of probabilities, as appropriate, in simple cases	Explicit use of the general formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ is not required.
• understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A) \times \mathrm{P}(B)$	
• calculate and use conditional probabilities in simple cases.	e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $\mathrm{P}(A\|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:	Notes and examples
• draw up a probability distribution table relating to a given situation involving a discrete random variable X, and calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$	
• use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models	Including the notations $\mathrm{B}(n, p)$ and $\mathrm{Geo}(p)$. $\mathrm{Geo}(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \ldots$.
• use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.	Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:	Notes and examples
• understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables	Sketches of normal curves to illustrate distributions or probabilities may be required.
• solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including – finding the value of $P(X > x_1)$, or a related probability, given the values of x_1, μ, σ. – finding a relationship between x_1, μ and σ given the value of $P(X > x_1)$ or a related probability	For calculations involving standardisation, full details of the working should be shown. e.g. $Z = \dfrac{(X - \mu)}{\sigma}$
• recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.	n sufficiently large to ensure that both $np > 5$ and $nq > 5$.

6 Probability & Statistics 2 (for Paper 6)

Knowledge of the content of Paper 5: Probability & Statistics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of calculus within the content for Paper 3: Pure Mathematics 3 will also be assumed.

6.1 The Poisson distribution

Candidates should be able to:	Notes and examples
• use formulae to calculate probabilities for the distribution $\text{Po}(\lambda)$	
• use the fact that if $X \sim \text{Po}(\lambda)$ then the mean and variance of X are each equal to λ	Proofs are not required.
• understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model	
• use the Poisson distribution as an approximation to the binomial distribution where appropriate	The conditions that n is large and p is small should be known; $n > 50$ and $np < 5$, approximately.
• use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate.	The condition that λ is large should be known; $\lambda > 15$, approximately.

6.2 Linear combinations of random variables

Candidates should be able to:	Notes and examples
• use, when solving problems, the results that – $\text{E}(aX + b) = a\text{E}(X) + b$ and $\text{Var}(aX + b) = a^2\text{Var}(X)$ – $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ – $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$ for independent X and Y – if X has a normal distribution then so does $aX + b$ – if X and Y have independent normal distributions then $aX + bY$ has a normal distribution – if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution.	Proofs of these results are not required.

6 Probability & Statistics 2

6.3 Continuous random variables

Candidates should be able to:	Notes and examples
• understand the concept of a continuous random variable, and recall and use properties of a probability density function	For density functions defined over a single interval only; the domain may be infinite, e.g. $\frac{3}{x^4}$ for $x \geqslant 1$.
• use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution.	Including location of the median or other percentiles of a distribution by direct consideration of an area using the density function. Explicit knowledge of the cumulative distribution function is not included.

6.4 Sampling and estimation

Candidates should be able to:	Notes and examples
• understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples	
• explain in simple terms why a given sampling method may be unsatisfactory	Including an elementary understanding of the use of random numbers in producing random samples. Knowledge of particular sampling methods, such as quota or stratified sampling, is not required.
• recognise that a sample mean can be regarded as a random variable, and use the facts that $\mathrm{E}(\overline{X}) = \mu$ and that $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$	
• use the fact that $(\overline{X})$ has a normal distribution if X has a normal distribution	
• use the Central Limit Theorem where appropriate	Only an informal understanding of the Central Limit Theorem (CLT) is required; for large sample sizes, the distribution of a sample mean is approximately normal.
• calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data	Only a simple understanding of the term 'unbiased' is required, e.g. that although individual estimates will vary the process gives an accurate result 'on average'.
• determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used	
• determine, from a large sample, an approximate confidence interval for a population proportion.	

6 Probability & Statistics 2

6.5 Hypothesis tests

Candidates should be able to:

- understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic
- formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using
  - direct evaluation of probabilities
  - a normal approximation to the binomial or the Poisson distribution, where appropriate
- formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used
- understand the terms Type I error and Type II error in relation to hypothesis tests
- calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.

Notes and examples

Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set.

4 Details of the assessment

Relationship between components

Candidates build their knowledge of the mathematics content as they progress through the course. Paper 1: Pure Mathematics 1 is the foundation for all other components. Paper 2: Pure Mathematics 2 and Paper 3: Pure Mathematics 3 build on the subject content for Paper 1: Pure Mathematics 1.

Paper 4: Mechanics and Paper 5: Probability & Statistics 1 components assume prior knowledge of the Paper 1: Pure Mathematics 1 content. The probability and statistics content in the qualification also builds sequentially through the components, so Paper 5: Probability & Statistics 1 is the foundation for studying Paper 6: Probability & Statistics 2.

The subject content assessed in the different examination components does not normally overlap. Only Paper 2 has a significant overlap with Paper 3, as the Pure Mathematics 2 subject content is largely a subset of the Pure Mathematics 3 subject content. Candidates may not take both Paper 2 and Paper 3 in the same examination series. Paper 2 and Paper 3 are taken in alternative routes through the qualification – Paper 2 is for AS Level only, and Paper 3 is for A Level.

Examination information

All components are assessed by written examinations which are externally marked. Sample assessment materials are available on our website at **www.cambridgeinternational.org** showing the question style and level of the examination papers.

Application of mathematical techniques

As well as demonstrating the appropriate techniques, candidates need to apply their knowledge in solving problems. Individual examination questions may involve ideas and methods from more than one section of the subject content for that component.

The main focus of examination questions will be the AS & A Level Mathematics subject content. However, in examination questions, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in section 3 of this syllabus.

Structure of the question paper

All questions in the examination papers are compulsory. An approximate number of questions for each paper is given in the Assessment overview in section 2 of this syllabus. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Answer space

Candidates answer on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

Degrees of accuracy

Candidates should give non-exact numerical answers correct to three significant figures (or one decimal place for angles in degrees) unless a different level of accuracy is specified in the question. To earn accuracy marks, candidates should avoid rounding figures until they have their final answer.

Additional materials for examinations

Candidates are expected to have the following equipment in examinations:

- a ruler
- a scientific calculator (see the following section).

Note: a protractor and a pair of compasses are **not** required.

A list of formulae and statistical tables (MF19) is supplied in examinations for the use of candidates. A copy of the list of formulae and tables is given for reference in section 5 of this syllabus. Note that MF19 is a combined formulae list for AS & A Level Mathematics (9709) and AS & A Level Further Mathematics (9231). Some formulae in the list are not needed for this syllabus, and are only for Further Mathematics (9231); these are listed in separate sections labelled Further Pure Mathematics, Further Mechanics, and Further Probability & Statistics.

Calculators

It is expected that candidates will have a calculator with standard 'scientific' functions available for use in all the examinations. Computers, graphical calculators and calculators capable of symbolic algebraic manipulation or symbolic differentiation or integration are not permitted. The General Regulations concerning the use of calculators are contained in the *Cambridge Handbook* at **www.cambridgeinternational.org/examsofficers**

Candidates are expected to show all necessary working; no marks will be given for unsupported answers from a calculator.

Mathematical notation

The list of mathematical notation that may be used in examinations for this syllabus is available on our website at **www.cambridgeinternational.org/9709**

Command words

The table below includes command words used in the assessment for this syllabus. The use of the command word will relate to the subject context.

Command word	What it means
Calculate	work out from given facts, figures or information
Describe	state the points of a topic / give characteristics and main features
Determine	establish with certainty
Evaluate	judge or calculate the quality, importance, amount, or value of something
Explain	set out purposes or reasons / make the relationships between things evident / provide why and/or how and support with relevant evidence
Identify	name/select/recognise
Justify	support a case with evidence/argument
Show (that)	provide structured evidence that leads to a given result
Sketch	make a simple freehand drawing showing the key features
State	express in clear terms
Verify	confirm a given statement/result is true

5 List of formulae and statistical tables (MF19)

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volume of cone or pyramid = $\frac{1}{3} \times$ base area $\times$ height

Area of curved surface of cone = $\pi r \times$ slant height

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d\,, \qquad S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}\,, \qquad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)\,, \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n\text{, where } n \text{ is a positive integer}$$

and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\text{, where } n \text{ is rational and } |x| < 1$$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \qquad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
x^n	nx^{n-1}
$\ln x$	$\dfrac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
uv	$v\dfrac{du}{dx} + u\dfrac{dv}{dx}$
$\dfrac{u}{v}$	$\dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathrm{d}x$	
x^n	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$	$(\lvert x\rvert < a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln\lvert\mathrm{f}(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

FURTHER PURE MATHEMATICS

Algebra

Summations:

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1), \qquad \sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1), \qquad \sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Maclaurin's series:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leqslant x \leqslant 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 < x < 1)$$

Trigonometry

If $t = \tan\frac{1}{2}x$ then:

$$\sin x = \frac{2t}{1+t^2} \qquad \text{and} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x \equiv 1, \qquad \sinh 2x \equiv 2\sinh x \cosh x, \qquad \cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad (x \geqslant 1)$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathrm{d}x$	
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$	$(\lvert x\rvert < \tfrac{1}{2}\pi)$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\tfrac{1}{2}x)\rvert$	$(0 < x < \pi)$
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	
$\operatorname{sech}^2 x$	$\tanh x$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$	$(x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$	

MECHANICS

Uniformly accelerated motion

$$v = u + at\,, \qquad s = \tfrac{1}{2}(u+v)t\,, \qquad s = ut + \tfrac{1}{2}at^2\,, \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}\,, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\tfrac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\tfrac{3}{8}r$ from centre

Hemispherical shell of radius r: $\tfrac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\tfrac{3}{4}h$ from vertex

PROBABILITY & STATISTICS

Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$\mathrm{E}(X) = \Sigma xp, \qquad \mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$$

For the binomial distribution $\mathrm{B}(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \qquad \mu = np, \qquad \sigma^2 = np(1-p)$$

For the geometric distribution Geo(p):

$$p_r = p(1-p)^{r-1}, \qquad \mu = \frac{1}{p}$$

For the Poisson distribution $\mathrm{Po}(\lambda)$

$$p_r = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}, \qquad \mu = \lambda, \qquad \sigma^2 = \lambda$$

Continuous random variables

$$\mathrm{E}(X) = \int x\,\mathrm{f}(x)\,\mathrm{d}x, \qquad \mathrm{Var}(X) = \int x^2\,\mathrm{f}(x)\,\mathrm{d}x - \{\mathrm{E}(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Central Limit Theorem:

$$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

$$\mathrm{N}\left(p, \frac{p(1-p)}{n}\right)$$

FURTHER PROBABILITY & STATISTICS

Sampling and testing

Two-sample estimate of a common variance:

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Probability generating functions

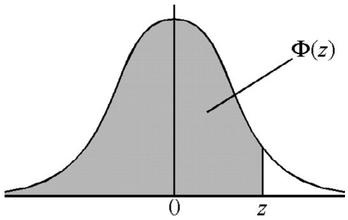
$$\mathrm{G}_X(t) = \mathrm{E}(t^X), \qquad \mathrm{E}(X) = \mathrm{G}'_X(1), \qquad \mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leqslant z).$$

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

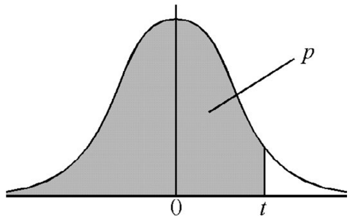
$$\mathrm{P}(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom, then, for each pair of values of p and ν, the table gives the value of t such that:

$P(T \leqslant t) = p$.

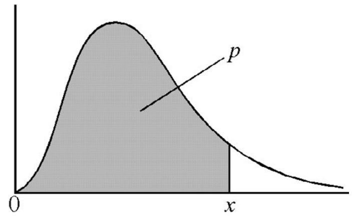


p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2-DISTRIBUTION

If X has a χ^2-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of x such that

$$P(X \leqslant x) = p.$$



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	0.0^31571	0.0^39821	0.0^23932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED-RANK TEST

The sample has size n.

P is the sum of the ranks corresponding to the positive differences.

Q is the sum of the ranks corresponding to the negative differences.

T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of T

	Level of significance			
One-tailed	0.05	0.025	0.01	0.005
Two-tailed	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK-SUM TEST

The two samples have sizes m and n, where $m \leqslant n$.

R_m is the sum of the ranks of the items in the sample of size m.

W is the smaller of R_m and $m(n + m + 1) - R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of W

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

6 What else you need to know

This section is an overview of other information you need to know about this syllabus. It will help to share the administrative information with your exams officer so they know when you will need their support. Find more information about our administrative processes at **www.cambridgeinternational.org/examsofficers**

Before you start

Previous study

We strongly recommend that learners starting this course should have studied a Cambridge IGCSE Mathematics (Extended) or Cambridge O Level course or the equivalent. See the introduction to section 3 of this syllabus for more details of expected prior knowledge.

Guided learning hours

We design Cambridge International AS & A Level syllabuses based on learners having about 180 guided learning hours for each Cambridge International AS Level and about 360 guided learning hours for a Cambridge International A Level. The number of hours a learner needs to achieve the qualification may vary according to local practice and their previous experience of the subject.

Availability and timetables

You can enter candidates in the June and November exam series. If your school is in India, you can enter your candidates in the March exam series. You can view the timetable for your administrative zone at **www.cambridgeinternational.org/timetables**

Private candidates can enter for this syllabus.

Combining with other syllabuses

Candidates can take this syllabus alongside other Cambridge International syllabuses in a single exam series. The only exceptions are:

- syllabuses with the same title at the same level.

Note that candidates can take AS & A Level Mathematics (9709) in the same exam series with AS & A Level Further Mathematics (9231).

Group awards: Cambridge AICE

Cambridge AICE (Advanced International Certificate of Education) is a group award for Cambridge International AS & A Level. It allows schools to offer a broad and balanced curriculum by recognising the achievements of learners who pass examinations in a range of different subjects.

Learn more about Cambridge AICE at **www.cambridgeinternational.org/aice**

Making entries

Exams officers are responsible for submitting entries to Cambridge International. We encourage them to work closely with you to make sure they enter the right number of candidates for the right combination of syllabus components. Entry option codes and instructions for submitting entries are in the *Cambridge Guide to Making Entries*. Your exams officer has a copy of this guide.

Exam administration

To keep our exams secure, we produce question papers for different areas of the world, known as 'administrative zones'. We allocate all Cambridge schools to one administrative zone determined by their location. Each zone has a specific timetable. Some of our syllabuses offer candidates different assessment options. An entry option code is used to identify the components the candidate will take relevant to the administrative zone and the available assessment options.

Support for exams officers

We know how important exams officers are to the successful running of exams. We provide them with the support they need to make your entries on time. Your exams officer will find this support, and guidance for all other phases of the Cambridge Exams Cycle, at **www.cambridgeinternational.org/examsofficers**

Retakes

Candidates can retake Cambridge International AS Level and Cambridge International A Level as many times as they want to. Cambridge International AS & A Levels are linear qualifications so candidates cannot re-sit individual components. Information on retake entries is in the *Cambridge Handbook* at **www.cambridgeinternational.org/examsofficers**

Equality and inclusion

We have taken great care to avoid bias of any kind in the preparation of this syllabus and related assessment materials. In compliance with the UK Equality Act (2010) we have designed this qualification to avoid any direct and indirect discrimination.

The standard assessment arrangements may present unnecessary barriers for candidates with disabilities or learning difficulties. We can put arrangements in place for these candidates to enable them to access the assessments and receive recognition of their attainment. We do not agree access arrangements if they give candidates an unfair advantage over others or if they compromise the standards being assessed.

Candidates who cannot access the assessment of any component may be able to receive an award based on the parts of the assessment they have completed.

Information on access arrangements is in the *Cambridge Handbook* at **www.cambridgeinternational.org/examsofficers**

Language

This syllabus and the related assessment materials are available in English only.

After the exam

Grading and reporting

Grades A*, A, B, C, D or E indicate the standard a candidate achieved at Cambridge International A Level, with A* being the highest grade.

Grades a, b, c, d or e indicate the standard a candidate achieved at Cambridge International AS Level, with 'a' being the highest grade.

'Ungraded' means that the candidate's performance did not meet the standard required for the lowest grade (E or e). 'Ungraded' is reported on the statement of results but not on the certificate. In specific circumstances your candidates may see one of the following letters on their statement of results:

- Q (pending)
- X (no result)
- Y (to be issued)

These letters do not appear on the certificate.

If a candidate takes a Cambridge International A Level and fails to achieve grade E or higher, a Cambridge International AS Level grade will be awarded if both of the following apply:

- the components taken for the Cambridge International A Level by the candidate in that series included all the components making up a Cambridge International AS Level
- the candidate's performance on the AS Level components was sufficient to merit the award of a Cambridge International AS Level grade.

On the statement of results and certificates, Cambridge International AS & A Levels are shown as General Certificates of Education, GCE Advanced Subsidiary Level (GCE AS Level) and GCE Advanced Level (GCE A Level).

'Cambridge International A Levels are the 'gold standard' qualification. They are based on rigorous, academic syllabuses that are accessible to students from a wide range of abilities yet have the capacity to stretch our most able.'

Mark Vella, Director of Studies, Auckland Grammar School, New Zealand

How students, teachers and higher education can use the grades

Cambridge International A Level

Assessment at Cambridge International A Level has two purposes.

- To measure learning and achievement.
  The assessment:
  - confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus, to the levels described in the grade descriptions.

- To show likely future success.
  The outcomes:
  - help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful
  - help students choose the most suitable course or career.

Cambridge International AS Level

Assessment at Cambridge International AS Level has two purposes.

- To measure learning and achievement.
  The assessment:
  - confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus.

- To show likely future success.
  The outcomes:
  - help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful
  - help students choose the most suitable course or career
  - help decide whether students part way through a Cambridge International A Level course are making enough progress to continue
  - guide teaching and learning in the next stages of the Cambridge International A Level course.

Grade descriptions

Grade descriptions are provided to give an indication of the standards of achievement candidates awarded particular grades are likely to show. Weakness in one aspect of the examination may be balanced by a better performance in some other aspect.

Grade descriptions for Cambridge International AS & Level Mathematics will be published after the first assessment of the A Level in 2020. Find more information at **www.cambridgeinternational.org/alevel**

Changes to this syllabus for 2020, 2021 and 2022

The syllabus has been reviewed and revised for first examination in 2020.

Significant additions to the syllabus content are indicated by vertical black lines either side of the text on pages 12–32 of this syllabus. Other changes, including removed content, are listed below.

In addition to reading the syllabus, teachers should refer to the updated specimen papers.

You are strongly advised to read the whole syllabus before planning your teaching programme.

Carry forward from 2019	• Candidates taking AS Level in 2019 can carry forward their result towards the full A Level with the revised syllabus in 2020.
Changes to availability of Mechanics 2 component	• Following consultation with schools and universities, from 2020 the Mechanics 2 component (formerly Paper 5) is no longer available in AS & A Level Mathematics. See below for information about alternative routes. Mechanics 2 content will be assessed at a higher level in the new AS & A Level Further Mathematics (9231) from 2020.
Changes to option routes through the qualification	• From 2020, there are two option routes to a full A Level Mathematics (9709): A Level candidates take Pure Mathematics 1 + Pure Mathematics 3, plus: EITHER Probability & Statistics 1 + Mechanics OR Probability & Statistics 1 + Probability & Statistics 2. • The numbering of assessment components from 2020 is as follows: Probability & Statistics 1 becomes Paper 5 (formerly Paper 6) Probability & Statistics 2 becomes Paper 6 (formerly Paper 7).
Changes to subject content	• Summary of overall changes to subject content The subject content has been updated following consultation, and given decimal numbering. Some topics have been removed or clarified and others added. The Mechanics 2 content has been removed from the syllabus. Notes and examples have been added to clarify the breadth and depth of content. The prior knowledge requirements have been clarified, to state that simple manipulation of surds and graphs of the form $y = kx^n$ are included. Summary of changes to Pure Mathematics 1 content by section • Section 1 Quadratics: linear inequalities content removed. • Section 2 Functions: transformations content added. • Section 3 Coordinate geometry: circles content added. • Vectors: This section has moved from Paper 1 to Paper 3. • Section 7 Differentiation: limits content added. Summary of changes to Pure Mathematics 2 content by section • Section 1 Algebra: content on sketching a modulus graph added. • Section 5 Integration: trapezium rule retained, but formula for it removed from List of formulae (MF19). *continued*

Changes to subject content continued	Summary of changes to Pure Mathematics 3 content by section • Section 1 Algebra: content on sketching a modulus graph added. • Section 4 Differentiation: the derivative of inverse tangent added. • Section 5 Integration: the idea of 'reverse differentiation' added. • Section 5 Integration: trapezium rule removed from Paper 3. • Section 7 Vectors: vector equations of planes removed; vector content moved from Paper 1 to Paper 3. Summary of changes to Mechanics (formerly Mechanics 1) by section • Section 3 Momentum: linear momentum and direct impact added. Summary of changes to Mechanics 2 • The entirety of the Mechanics 2 content has been removed from A Level Mathematics (9709) and will be part of the new AS & A Level Further Mathematics (9231) from 2020. Summary of changes to Probability & Statistics 1 content by section • Notes added on algebra content and probability notation. • Section 4 Discrete random variables: geometric distribution added. Summary of changes to Probability & Statistics 2 content by section • There are no significant changes to this component. Other changes in the syllabus document • Key concepts for the syllabus have been introduced.
Changes to assessment (including changes to specimen papers)	• Changes to duration of examinations The durations of the Paper 1 examination and the Paper 3 examination have increased by 5 minutes to 1 hour 50 minutes each. • Changes to AOs and aims The single assessment objective (AO) has been divided into two AOs. There is no fundamental change in meaning. The weighting of the AOs has been identified by component. The aims have been clarified. *continued*

Changes to assessment (including changes to specimen papers) continued	• Changes to how question papers are presented Questions will no longer remind candidates to show their working, as a new front cover statement is included. The instructions for candidates on the front cover of examination papers have been amended to add: 'You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.' 'If additional space is required, you should use the lined page at the end of the booklet; the question number or numbers must be clearly shown.' All part questions from 2020 are numbered using the labelling (a), (b), (c), whether the parts are dependent or independent. Roman numerals will only be used for labelling further divisions within a part e.g. (a)(i). Questions will not necessarily be in ascending order of tariff. • Changes to the List of formulae and statistical tables The MF9 List of formulae and statistical tables is being replaced from 2020 with a new list, MF19, which combines formulae for AS & A Level Mathematics (9709) and AS & A Level Further Mathematics (9231). The MF19 list includes new formulae for additional topics introduced, and the trapezium rule is removed. AS & A Level Mathematics candidates will not need to use formulae from the sections with 'Further' in the heading. • Changes to the list of mathematical notation The list of mathematical notation that may be used in examinations for this syllabus has been updated and is available on our website at **www.cambridgeinternational.org/9709** The specimen materials have been revised to reflect the new assessment structure and syllabus content and these are available on our website at **www.cambridgeinternational.org** The syllabus and specimen papers use our new name, Cambridge Assessment International Education.

Any textbooks endorsed to support the syllabus for examination from 2020 are suitable for use with this syllabus.

'While studying Cambridge IGCSE and Cambridge International A Levels, students broaden their horizons through a global perspective and develop a lasting passion for learning.'

Zhai Xiaoning, Deputy Principal, The High School Affiliated to Renmin University of China

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