SEKOLAH BUKIT SION

High School Examination

STUDENT NAME			
EXAM NUMBER		CLASS	

ADDITIONAL MATHEMATICS

Year 10

6 Dec 2021

120 minutes

Students answer on the Question Paper/File Paper.
Additional Materials: Scientific Calculator

READ THESE INSTRUCTIONS FIRST.

Write your name and class on all the work you hand in.
Write in dark blue or black pen in all your working and answers.
Use an HB pencil for any constructions and in reading diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
For questions with asterisk, choose/answer only one. Do not answer both.
Give non-exact numerical answers correct to **3 significant figures,** unless the level of accuracy specified in the question.
The use of an electronic calculator is expected, where expected.
You are reminded of the need for clear presentation in your working and answers.

At the end of the examination, you are given 10 minutes to scan all your work and submit/attach **a PDF version** of your work with the filename ***NameClass_Sem1Exam***
Make sure to submit your work on time. Late submissions may not be accepted.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

1 The function f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = x^3 \qquad \text{and} \qquad g(x) = x + 2$$

Express each of the following as a composite function using only f, g, f^{-1} and/or g^{-1}:

(a) $x \to x^3 + 2$ **[1]**

(b) $x \to x^3 - 2$ **[1]**

(c) $x \to (x + 2)^{1/3}$ **[1]**

2

(a) Solve the equation $2 = |e^{-x} - 3|$ **[3]**

(b) Solve: $2 + \ln(4 - x) = 0$. **[2]**

3

The polynomial $P(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, leaves a remainder of 10 when divided by $(2x - 1)$. Given that $(x + 2)$ is a factor of $P(x)$,

(a) find the value of a and b. **[4]**

(b) Explain why $P(x) = 0$ has only 1 root. **[4]**

4* **CHOOSE/ANSWER only 1.** **[5]**

(a) $\log_2 \sqrt{5x + 1} = \log_4(x - 2) + \log_2 4$

(b) $\lg(4^x - 10) - x \lg 2 = \lg 3$

5* **CHOOSE/ANSWER only 1.** **[3]**

(a) Given that $3^5 \div \frac{1}{27} = \frac{1}{3^{2k}}$, solve for 2^k.

(b) Given that $2^{2x-3} = \frac{1}{4^{x-1}}$, solve for x.

6 Solve the following equations.

(a) $2(3^x) + 3^{-x} = 3$ **[3]**

(b) $2\sqrt{3-2x} = x + 1$ **[3]**

7

(a) Describe the nature of the roots of $7x^2 + 4x - 1 = 0$. **[2]**

(b)* **CHOOSE/ANSWER only 1.** **[4]**

(i) Find the range of values of k such that $kx^2 - kx + 3x + 3 - k = 0$ has no real roots.

(ii) Find the **smallest positive integer** p, such that $4(px - 5) = x^2$ has real roots.

8

(a) Determine whether $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ or not. **[1]**

(b) Factorise $x^3 - 7x^2 + 2x + 40$ completely. **[3]**

---

9 The straight line $2y = 3x - 16$ intersects the curve $2x^2 - 19x + 2y + 40 = 0$ at the points A and B.

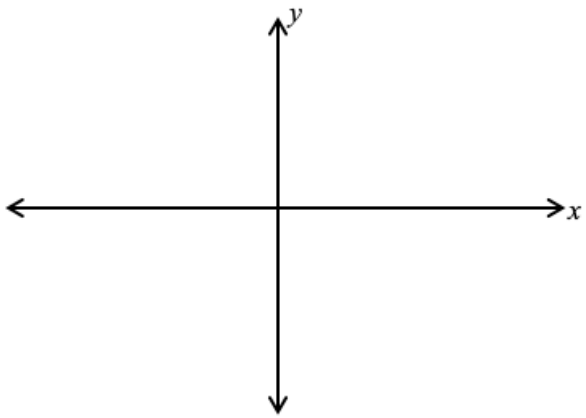
Find the distance of AB, giving your answer in simplified surd form when necessary. **[5]**

---

10 Given that $f(x) = 3x - 7$ and $g(x) = \frac{12}{x-2}$, $x \neq 2$.

(a) Find the values of x for which $fg(x) = x$. **[3]**

(b) Sketch the graphs of f and f^{-1} on the same diagram, giving the coordinates of the points of intersection of each graph with the axes. **[3]**



---

11 Show that $(2 + \sqrt{7})^2 - \frac{18}{3-\sqrt{7}} = c + d\sqrt{7}$, where c and d are integers. **[4]**

---

12 Functions f and g are defined for $x \in \mathbb{R}$ by

$f : x \rightarrow e^x$ and $g : x \rightarrow 2x - 3$

(a) Solve the equation $fg(x) = 7$. [2]

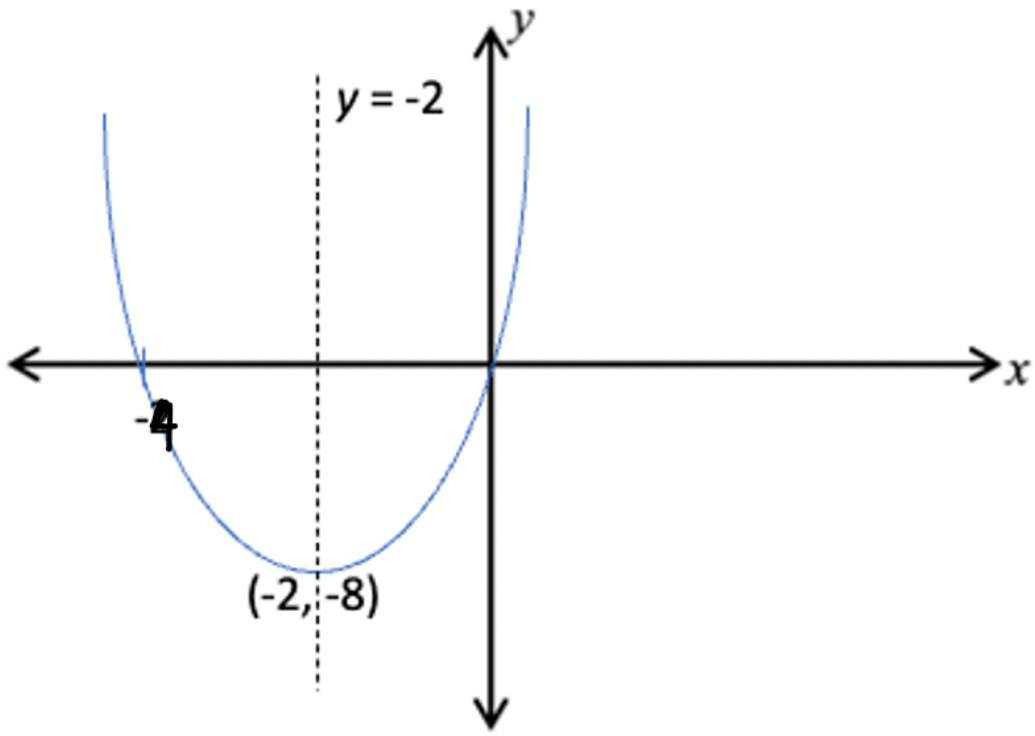
Function h is defined as gf.

(b) Express h in terms of x and state its range. [2]

(c) Express h^{-1} in terms of x. [2]

---

13 Using symmetry, Ariel sketched the graph of $y = x(4 - x)$ on the axes provided as shown. He also indicated the turning point coordinates and intercepts, equation of line of symmetry.



Discuss four (4) of Ariel's mistakes. [4]

14

(a) Express $x^2 + 6x - 11$ in the form $(x + b)^2 + c$.
Hence, write down the coordinates of the turning point of $f(x) = x^2 + 6x - 11$. [4]

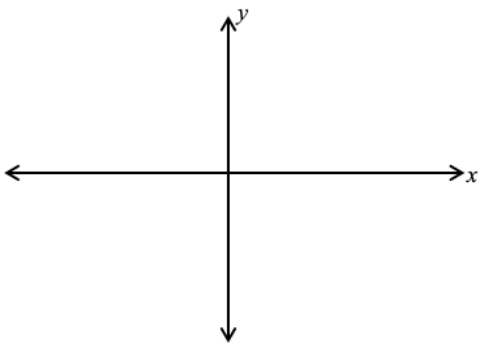
(b) Write down the range of $f(x) = x^2 + 6x - 11$ for domain $1 \leq x \leq 6$. [2]

(c) For the function f to have an inverse in the domain $x \geq p$, find the smallest value of p. [1]

(d) Solve for the range of values of x if $x^2 + 6x - 11 > 5$. [3]

(e) Sketch the graph of $y = |x^2 + 6x - 11|$ on the axes provided,

indicating the turning point coordinates and intercepts. [3]



Hence, write down the number of solutions of $|x^2 + 6x - 11| = 20$. [1]